

Ch 10 Exponential and Logarithmic Functions

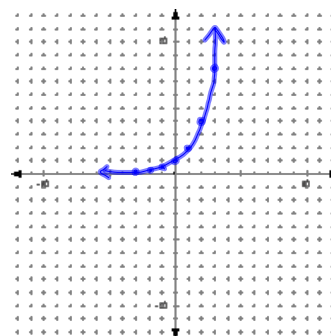
10-1 Exponential Functions

Exponential function--function with a variable in the exponent

ex

$$y = 2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



Form

$$y = a b^x$$

$$a \neq 0$$

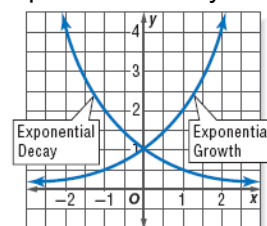
$$b > 0$$

$$b \neq 1$$

$$y = \frac{1}{2} x$$

graph on calculator

There are 2 types of exponential functions:
exponential growth and exponential decay.



Key Concept

Exponential Growth and Decay

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

Characteristics (p524)

1. The function is continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x -axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y -intercept is a .
6. The graphs of $y = ab^x$ and $y = a(\frac{1}{b})^x$ are reflections across the y -axis.

Write an exponential function whose graph passes through the given points.

$(0, 5)$ $(2, 45)$

$$\begin{aligned}
 y &= ab^x \\
 5 &= a \cdot b^0 \\
 5 &= a \cdot 1 \\
 5 &= a \\
 y &= 5 \cdot b^x \\
 45 &= 5 \cdot b^2 \\
 9 &= b^2 \\
 3 &= b \\
 y &= 5 \cdot 3^x
 \end{aligned}$$

Write an exponential function whose graph passes through the given points.

$(0, 4)$ $(3, \frac{1}{2})$

$$\begin{aligned}
 y &= a \cdot b^x \\
 4 &= a \cdot b^0 \\
 4 &= a \\
 y &= 4b^x \\
 \frac{1}{2} &= 4b^3 \\
 \frac{1}{8} &= b^3 \\
 \frac{1}{2} &= b \\
 y &= 4\left(\frac{1}{2}\right)^x
 \end{aligned}$$

Solving exponential equations.

ex

$$8^x = \frac{1}{4}$$

$$\begin{aligned}
 8^{-\frac{2}{3}} &= \frac{1}{4} \\
 \frac{1}{8^{\frac{2}{3}}} &= \frac{1}{4} \\
 2^{3x} &= 2^{-2} \\
 \frac{1}{4} &= \frac{1}{4} \checkmark
 \end{aligned}$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

Must check

ex

$$5^{4-t} = 25^{t-1}$$

$$5^{4-t} = (5^2)^{t-1}$$

$$4-t = 2t-2$$

$$6 = 3t$$

$$2 = t \quad \checkmark$$

$$\text{ex } 2^5 = 32$$

$$\sqrt[3]{32} = 2^x$$

$$2^{\frac{5}{3}} = 2^x$$

$$\frac{5}{3} = x$$

ex

$$9^{\frac{x}{2}}$$

$$9^{x-3} = 27$$

$$(3^2)^{x-3} = 3^3$$

$$2x-6 = 3$$

$$2x = 9$$

$$x = 4.5$$

Do:

$$1. 4^{x-2} = 64^x$$

$$(4)^{3x}$$

$$x-2 = 3x$$

$$x = -1$$

$$2. 4^{2x+5} = 16^{x+1}$$

$$4^{2x+5} = (4^2)^{x+1}$$

$$2x+5 = 2x+2$$

$$5 \neq 2$$

$$\emptyset$$

Exponential Inequalities

ex

$$3^x < \frac{1}{27}$$

$$3^x < 3^{-3}$$

$$x < -3$$

ex

$$5^x > \sqrt{125} \quad \sqrt{5^3}$$

$$5^x > 5^{3/2}$$

$$x > \frac{3}{2}$$

ex

$$25^{2x} < 5^{(x+6)}$$

1. **OPEN ENDED** Give an example of a value of b for which $y = b^x$ represents exponential decay. $b = .2$

2. **Identify** each function as *linear*, *quadratic*, or *exponential*.

a. $y = 3x^2$

b. $y = 4(3)^x$

c. $y = 2x + 4$

d. $y = 4(0.2)^x + 1$

Q

E

L

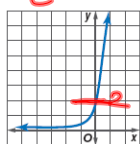
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Match each function with its graph.

3. $y = 5^x$

C

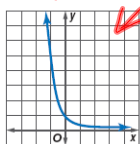
a.



4. $y = 2(5)^x$

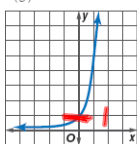
A

b.



5. $y = \left(\frac{1}{5}\right)^x$

c.



HW

p528-529

21, 25, 27-29, 33, 35, 39-53odd